

Laws of logic: Bitwise Operators

Zero is always represented by all 0's. For signed values, -1 is always represented by all ones.

A. Negation, Associativity, Commutativity

Negation:

$$\sim(\sim p) = p$$

Associative:

$$(p \mid q) \mid r = p \mid (q \mid r) \quad (p \& q) \& r = p \& (q \& r)$$

Commutative:

$$p \mid q = q \mid p \quad p \& q = q \& p$$

B. De Morgan's laws

$$\sim(p \mid q) = \sim p \& \sim q \quad \sim(p \& q) = \sim p \mid \sim q$$

$$\sim(\sim p \mid \sim q) = p \& q \quad \sim(\sim p \& \sim q) = p \mid q$$

C. Identity laws:

$$p \mid 0 = p \quad p \& 0 = 0$$

$$p \mid -1 = -1 \quad p \& -1 = p$$

D. Inverse Laws:

$$p \mid \sim p = -1 \quad p \& \sim p = 0$$

E. Idempotent Laws:

$$p \mid p = p \quad p \& p = p$$

$$(p \& q) \mid (p \& q) = p \& q \quad (p \mid q) \& (p \mid q) = (p \mid q)$$

F. Distributive Laws:

$$p \& q \mid r = (p \mid r) \& (q \mid r) \quad (p \mid r) \& (q \mid r) = (p \& q) \mid r$$

$$(p \mid q) \& r = (p \& r) \mid (q \& r) \quad (p \& r) \mid (q \& r) = (p \mid q) \& r$$

G. Absorption Laws:

$$p \& (p \mid q) = p \quad p \mid (p \& q) = p$$

Laws of logic: Logical Operators

The value “false” is represented by 0, “true” is represented by 1.

A. Negation, Associativity, Commutativity

Negation:

$$!(\textcolor{blue}{!}\textcolor{blue}{p}) = \textcolor{blue}{p}$$

Associative:

$$(\textcolor{blue}{p} \parallel \textcolor{green}{q}) \parallel r = \textcolor{blue}{p} \parallel (\textcolor{green}{q} \parallel r) \quad (\textcolor{blue}{p} \&& \textcolor{green}{q}) \&& r = \textcolor{blue}{p} \&& (\textcolor{green}{q} \&& r)$$

Commutative:

$$\textcolor{blue}{p} \parallel \textcolor{green}{q} = \textcolor{green}{q} \parallel \textcolor{blue}{p} \quad \textcolor{blue}{p} \&& \textcolor{green}{q} = \textcolor{green}{q} \&& \textcolor{blue}{p}$$

B. De Morgan's laws:

$$!(\textcolor{blue}{p} \parallel \textcolor{green}{q}) = \textcolor{blue}{!}\textcolor{blue}{p} \&& \textcolor{blue}{!}\textcolor{green}{q} \quad !(\textcolor{blue}{p} \&& \textcolor{green}{q}) = \textcolor{blue}{!}\textcolor{blue}{p} \parallel \textcolor{blue}{!}\textcolor{green}{q}$$

$$!(\textcolor{blue}{!}\textcolor{blue}{p} \parallel \textcolor{blue}{!}\textcolor{green}{q}) = \textcolor{blue}{p} \&& \textcolor{green}{q} \quad !(\textcolor{blue}{!}\textcolor{blue}{p} \&& \textcolor{blue}{!}\textcolor{green}{q}) = \textcolor{blue}{p} \parallel \textcolor{green}{q}$$

C. Identity laws:

$$\begin{array}{ll} \textcolor{blue}{p} \parallel 0 = \textcolor{blue}{p} & \textcolor{blue}{p} \&& 0 = 0 \\ \textcolor{blue}{p} \parallel 1 = 1 & \textcolor{blue}{p} \&& 1 = \textcolor{blue}{p} \end{array}$$

D. Inverse Laws:

$$\textcolor{blue}{p} \parallel \textcolor{blue}{!}\textcolor{blue}{p} = 1 \quad \textcolor{blue}{p} \&& \textcolor{blue}{!}\textcolor{blue}{p} = 0$$

E. Idempotent Laws:

$$\begin{array}{ll} \textcolor{blue}{p} \parallel \textcolor{blue}{p} = \textcolor{blue}{p} & \textcolor{blue}{p} \&& \textcolor{blue}{p} = \textcolor{blue}{p} \\ (\textcolor{blue}{p} \&& \textcolor{green}{q}) \parallel (\textcolor{blue}{p} \&& \textcolor{green}{q}) = \textcolor{blue}{p} \&& \textcolor{green}{q} & (\textcolor{blue}{p} \parallel \textcolor{green}{q}) \&& (\textcolor{blue}{p} \parallel \textcolor{green}{q}) = (\textcolor{blue}{p} \parallel \textcolor{green}{q}) \end{array}$$

F. Distributive Laws:

$$\begin{array}{ll} (\textcolor{blue}{p} \&& \textcolor{green}{q}) \parallel r = (\textcolor{blue}{p} \parallel r) \&& (\textcolor{green}{q} \parallel r) & (\textcolor{blue}{p} \parallel r) \&& (\textcolor{green}{q} \parallel r) = (\textcolor{blue}{p} \&& \textcolor{green}{q}) \parallel r \\ (\textcolor{blue}{p} \parallel \textcolor{green}{q}) \&& r = (\textcolor{blue}{p} \&& r) \parallel (\textcolor{green}{q} \&& r) & (\textcolor{blue}{p} \&& r) \parallel (\textcolor{green}{q} \&& r) = (\textcolor{blue}{p} \parallel \textcolor{green}{q}) \&& r \end{array}$$

G. Absorption Laws:

$$\textcolor{blue}{p} \&& (\textcolor{blue}{p} \parallel \textcolor{green}{q}) = \textcolor{blue}{p} \quad \textcolor{blue}{p} \parallel (\textcolor{blue}{p} \&& \textcolor{green}{q}) = \textcolor{blue}{p}$$