

Laws of logic

A. Negation, Associativity, Commutativity

Negation:

$$\sim(\sim p) = \sim\sim p = p$$

Associative:

$$(p + q) + r = p + (q + r) \quad (p \wedge q) \wedge r = p \wedge (q \wedge r)$$

Commutative:

$$p + q = q + p \quad p \wedge q = q \wedge p$$

B. De Morgan's laws

$$\begin{aligned}\sim(p + q) &= (\sim p) \wedge (\sim q) \\ \sim(p \wedge q) &= (\sim p) + (\sim q)\end{aligned}$$

$$\begin{aligned}\sim(\sim p \wedge \sim q) &= (p + q) \\ \sim(\sim p + \sim q) &= (p \wedge q)\end{aligned}$$

C. Identity laws:

$$\sim\text{true} = \text{false} \quad \sim\text{false} = \text{true}$$

$$\begin{aligned}p + \text{false} &= p \\ p \wedge \text{true} &= p\end{aligned}$$

$$\begin{aligned}p + \text{true} &= \text{true} \\ p \wedge \text{false} &= \text{false}\end{aligned}$$

D. Inverse Laws:

$$p + \sim p = \text{true} \quad p \wedge \sim p = \text{false}$$

E. Idempotent Laws:

$$\begin{aligned}p + p &= p \\ (p \wedge q) + (p \wedge q) &= (p \wedge q)\end{aligned}$$

$$\begin{aligned}p \wedge p &= p \\ (p + q) \wedge (p + q) &= (p + q)\end{aligned}$$

F. Distributive Laws:

$$\begin{aligned}(p \wedge q) + r &= (p + r) \wedge (q + r) \\ (p + q) \wedge r &= (p \wedge r) + (q \wedge r)\end{aligned}$$

$$\begin{aligned}p + (q \wedge r) &= (p + q) \wedge (p + r) \\ p \wedge (q \wedge r) &= (p \wedge q) + (p \wedge r)\end{aligned}$$

Factoring Form of the Distributive Law:

Left factoring:

$$\begin{aligned}(p \wedge q) + (p \wedge r) &= p \wedge (q + r) \\ (p + q) \wedge (p + r) &= p + (q \wedge r)\end{aligned}$$

Right factoring:

$$\begin{aligned}(p \wedge r) + (q \wedge r) &= (p + q) \wedge r \\ (p + r) \wedge (q + r) &= (p \wedge q) + r\end{aligned}$$

G. Absorption Laws:

$$\begin{aligned}p \wedge (p + q) &= p \\ (p + q) \wedge p &= p\end{aligned}$$

$$\begin{aligned}p + (p \wedge q) &= p \\ (p \wedge q) + p &= p\end{aligned}$$