

Laws of logic

A. Negation, Associativity, Commutativity

Negation:

$$\sim(\sim p) = \sim\sim p = p$$

Associative:

$$(p + q) + r = p + (q + r)$$

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$

Commutative:

$$p + q = q + p$$

$$p \wedge q = q \wedge p$$

B. De Morgan's laws

$$\sim(p + q) = (\sim p) \wedge (\sim q)$$

$$\sim(\sim p \wedge \sim q) = (p + q)$$

$$\sim(p \wedge q) = (\sim p) + (\sim q)$$

$$\sim(\sim p + \sim q) = (p \wedge q)$$

C. Identity laws:

$$\sim \text{true} = \text{false}$$

$$\sim \text{false} = \text{true}$$

$$p + \text{false} = p$$

$$p + \text{true} = \text{true}$$

$$p \wedge \text{true} = p$$

$$p \wedge \text{false} = \text{false}$$

D. Inverse Laws:

$$p + \sim p = \text{true}$$

$$p \wedge \sim p = \text{false}$$

E. Idempotent Laws:

$$p + p = p$$

$$p \wedge p = p$$

$$(p \wedge q) + (p \wedge q) = (p \wedge q)$$

$$(p + q) \wedge (p + q) = (p + q)$$

F. Distributive Laws:

$$(p \wedge q) + r = (p + r) \wedge (q + r)$$

$$p + (q \wedge r) = (p + q) \wedge (p + r)$$

$$(p + q) \wedge r = (p \wedge r) + (q \wedge r)$$

$$p \wedge (q + r) = (p \wedge q) + (p \wedge r)$$

Factoring Form of the Distributive Law:

Left factoring:

$$(p \wedge q) + (p \wedge r) = p \wedge (q + r)$$

$$(p + q) \wedge (p + r) = p + (q \wedge r)$$

Right factoring:

$$(p \wedge r) + (q \wedge r) = (p + q) \wedge r$$

$$(p + r) \wedge (q + r) = (p \wedge q) + r$$

G. Absorption Laws:

$$p \wedge (p + q) = p$$

$$p + (p \wedge q) = p$$

$$(p + q) \wedge p = p$$

$$(p \wedge q) + p = p$$